BUSINESS STATISTICS

CHAPTER 3

MEASURES OF LOCATION OR CENTRAL TENDENCY

- We use the same term indiscriminately in speaking of, for example, the "average Ghanaian", the "average worker", the "average student", etc.
- Synonyms for the term in its popular usages are such expressions as "typical", usual", "representative, "normal", and "expected".
- Whatever may be the specific meanings of the term "average", it is reasonably clear that, it is a single value that describes a set of data.
- An "**AVERAGE"** is a value that is typical or representative of a set of data. Since such typical values *average* tend to lie centrally within a set of data arranged according to magnitude, averages are also called *measures of central tendency.*

Measure of Central Tendency defined

- It is a single value that summarizes a set of data. It locates the centre of the values.
- For the sake of this study, the measures of central tendency to consider are;

1. *arithmetic mean*
2. *weighted mean*
3. *Mode*
4. *median*
5. *geometric mean*

ARITHMETIC MEAN

Under this measure of central tendency, there are two main calculations.

- Arithmetic mean for the **population**
- Arithmetic mean for the **sample**

POPULATION MEAN

- Used for studies that involve all the values in a population.
- Eg. If we report that the mean age of all professors at KNUST is 55, this is an example of a population because we had the ages of all professors at KNUST.
- It is thus calculated as the sum of all the values in the population divided by the number of values in the population.
- $$\textbf{Population mean} = \frac{\textbf{Sum of all the values in the population}}{\textbf{Number of values in the population}}$$
- $$\boldsymbol{\mu} = \frac{\sum x}{N}$$
- where,
- μ = population mean
- N = number of items in the population.
- x = represents any particular value
- Σ = sum of the values.

Example 3.1

Marks obtained by 10 students in Business Mathematics are given below:

46 52 75 40 70 43 40 65 35 48

Calculate the arithmetic mean.

Solution

$$\mu = \frac{\sum x}{N}$$

$$\mu = \frac{46 + 52 + 75 + 40 + 70 + 43 + 40 + 65 + 35 + 48}{10}$$

$$= \frac{514}{10} = \text{51.4 marks}$$

Example 3.2

- There are 12 automobile companies in United States. Listed below is the number of patents granted by the United States government to each company last year.
- Is the information a sample or a population?

COMPANY	NO. OF PATENTS	COMPANY	NO. OF PATENTS
General Motors	511	**Chrysler**	97
Nissan	385	**Porshe**	50
Daumier- Benz	275	**Mitsubishi**	36
Toyota	257	**Volvo**	23
Honda	249	**B.M.W**	13
Ford	234	**Mazda**	210

- What is the arithmetic mean number of patents granted?

SAMPLE MEAN

We select a sample from the population in order to find out something about a specific characteristic of the population.

Sample mean ($\bar{x}$) = $\frac{\sum x}{n}$

$\bar{x}$ = x bar

n = number in the sample.

The mean of a sample or any other measure based on sample data, is called a statistic.

Example 3.3

- In the table below, a random sample of six bonds revealed the following:

Issue Interest Rate	**(%)**
Australian government bonds	9.50
Belgian government bonds	7.25
Canadian government bonds	6.50
French government “B-TAN”	4.75
Italian government bonds	12.00
Spanish government bonds	8.30

What is the arithmetic mean interest rate on this sample of long term obligations?

WEIGHTED MEAN

- The weighted mean is a special type of the arithmetic mean.
- It occurs when there are several observations of the same value which might occur if the data have been grouped into a frequency distribution.
- In general the weighted mean of a set of numbers designated X_1, X_2, X_3, X_n with the corresponding weights.

$$Weighted\ Mean\ \bar{X}_w = \frac{W_1 X_1 + W_2 X_2 + W_3 X_3 + \ldots + W_n X_n}{W_1 + W_2 + W_3 + \ldots + W_n}$$

$$\bar{X}_w = \frac{\sum(WX)}{\sum W}$$

Example 3.4

The Quality Construction Company pays its hourly employees GH¢6500, GH¢7500 or GH¢8500 per hour. There are 26 hourly employees. 14 are paid at the GH¢6500 rate, 10 at the GH¢7500 and 2 at the GH¢8500.

What is the mean hourly rate paid the 26 employees?

Example 3.5

In a company having 80 employees, 60 earn \$10.00 per hour and 20 earn \$13.00 per hour.

a) Determine the mean earning per hour

b) Do you believe the mean hourly wage to be typical?

THE MEDIAN

- Median is the central value of the variable when the values are arranged in ascending or descending order of magnitude.
- Let **N** be the number of observations in array, then

a. when n is odd, the middle value i.e. ½(n + 1)th value gives the median (or median class)

b. When is even there are two middle values. The arithmetic mean of these two values gives the median.

EXAMPLE 3.7

Find the median from the following data:

2 30 12 25 20 8 10 4 15

EXAMPLE 3.8

According to the census of 1981, the following are the population figures, in thousands of 10 cities:

2000, 1180, 1785, 1500, 560, 782, 1300, 385, 1123, 222.

Find the median

THE MODE

The mode (or modal class) of a set of observations is the value (or class) with the highest frequency (the most common value or class).

A mode may not exist, if it does, it may not be unique.

EXAMPLE 3.9

Determine the mode from the following data:

25, 15, 23, 40, 27, 25, 23, 25, and 20

EXAMPLE 3.10

Find the mode of 12, 12, 13, 14, 14, 15

THE GEOMETRIC MEAN

- The geometric mean is useful in finding the average of percentages, ratios, indexes or growth rates.
- It has a wide application in business and economics because we are often interested in finding the percentage change in sales, salaries or economic figures, such as the Gross National Product (G.N.P.).
- The Geometric mean of a set of n positive numbers is defined as the nth root of the product of n values. The formula for the geometric mean is written:

$$\text{GM} = \sqrt[n]{(X_1)(X_2)(X_3)\ldots.(X_n)}$$

THE GEOMETRIC MEAN

- Note: The geometric mean will always be less than or equal to (never more than) the arithmetic mean.
- Note also that all the data values must be positive to determine the geometric mean.

EXAMPLE 3.11

- The profits earned by Consar Construction Company on four recent projects were 3 percent, 2 percent, 4 percent and 6 percent. What is the geometric mean project?

APPLICATION 3.1

- A second application of the geometric mean is to find an average percent increase over a period of time.
- **For example if you earned GH¢30,000 a year in 1990 and GH¢50,000 in the year 2000. What is your annual rate of increase over the period?**
- The rate of increase is determined from the following formula:
- $$GM = \sqrt[n]{\frac{Value\ at\ the\ end\ of\ period}{Value\ at\ the\ beginning\ of\ period}} - 1$$

APPLICATION 3.2

Suppose you receive a 5 percent increase in salary this year and a 15 percent increase next year. What is the average percent increase?

Example 3.12

The population in Ghana in 1988 was 2 persons, by 1998 it was 22. What is the average annual rate of percentage increase during are 10 years between 1988 and 1998 so n = 10.

The Mean, Median and Mode of Grouped Data

- **ARITHMETIC MEAN**

To approximate the arithmetic mean of the data organized into a frequency distribution, we begin by assuming the observations in each class are represented by the midpoint of the class. The mean of a sample of data organized in a frequency distribution is computed by

METHOD 1 $$x = \frac{\sum fx}{\sum f}$$

x is the mid-value or midpoint of each class

f is the frequency in each class

fx is the frequency in each class times the midpoint of the class

Σ*fx* is the sum of these products

n is the total number of frequencies

Example 3.13

Class Limits	*Class Boundaries*	*x*	*f*	*fx*
3 – 7	2.5 – 7.5		20	
8 – 12	7.5 – 12.5		43	
13 – 17	12.5 – 17.5		75	
18 – 22	17.5 – 22.5		67	
23 – 27	22.5 – 27.5		72	
28 – 32	27.5 – 32.5		45	
33 – 37	32.5 – 37.5		39	
38 – 42	37.5 – 42.5		9	
43 – 47	42.5 – 47.5		8	
48 – 52	47.5 – 52.5		6	
			$\sum f =$	$\sum f x =$

METHOD 2

- If A is the assumed mean and the deviations are d (d = X - A) , then the mean is

$$\mathbf{x} = \frac{\sum \boldsymbol{fx}}{\sum \boldsymbol{f}} = \frac{\sum \boldsymbol{f}(\boldsymbol{A}+\boldsymbol{d})}{\sum \boldsymbol{f}}$$

$$d + A = x$$

$$= \frac{\boldsymbol{A}\sum \boldsymbol{f}}{\sum \boldsymbol{f}} + \frac{\sum \boldsymbol{fd}}{\sum \boldsymbol{f}}$$

$$= A + \frac{\sum \boldsymbol{fd}}{\sum \boldsymbol{f}}$$

Example 3.14 A= 30

Class Limits	***Class Boundaries***	***x***	***f***	***d = x -30***	***fd***
3 – 7	2.5 – 7.5		20		
8 – 12	7.5 – 12.5		43		
13 – 17	12.5 – 17.5		75		
18 – 22	17.5 – 22.5		67		
23 – 27	22.5 – 27.5		72		
28 – 32	27.5 – 32.5		45		
33 – 37	32.5 – 37.5		39		
38 – 42	37.5 – 42.5		9		
43 – 47	42.5 – 47.5		8		
48 – 52	47.5 – 52.5		6		
			$\sum f =$		$\sum f\, d =$

$$x = A + \frac{\sum fd}{\sum f}$$

$$= 30 + \left(\frac{-2990}{384}\right)$$

$= 30 – 7.8$

$=$ **22.2**

METHOD 3

- This is for a grouped data with equal class intervals, C
- The coded values are defined by

$$\mu = \frac{deviation}{class\ interval} = \frac{d}{c}$$

But $d = x - A$ $\quad d = \mu c$

$$\mu = \frac{x - A}{c}$$

$$= A + \frac{\sum fd}{\sum f}$$

$$= A + \frac{\sum f\mu c}{\sum f}$$

$$= A + \left(\frac{\sum f\mu}{\sum f}\right) c$$

Example 3.15

let c = 5

x	f	$d = x - A$	$\mu = \frac{x - A}{c}$	$f\mu$
5	20			
10	43			
15	75			
20	67			
25	72			
30	45			
35	39			
40	9			
45	8			
50	6			
	$\sum f =$			$\sum f\mu =$

EXERCISE 3.1

- Find the mean of the distribution below using the 3 methods:

A = 158

WEIGHT	FREQUENCY
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 – 171	4
172 - 180	2
	TOTAL = 40

EXERCISE 3.2

For a frequency distribution in statistics for 100 students, the arithmetic average was found to be 50. Later on it was discovered that marks 48 were misread as 84. Find the correct mean.

EXERCISE 3.3

Calculate the missing value from the following data if average mean= 20

NO OF TABLETS	NO OF PERSONS CURED
4 – 8	11
8 – 12	13
12 – 16	16
16 – 20	14
20 – 24	**?**
24 – 28	9
28 – 32	17
32 – 36	6
36 - 40	4

EXERCISE 3.4

The mean weight of 150 students in a certain class is 60kg. The mean weight of boys in the class is 70kg and that of the girls is 55 kg.

Find the number of boys and the number of girls in the class.

MEDIAN FROM A GROUPED DATA

- Median = L + $\left[\frac{\frac{n}{2}-Cfm}{fm}\right] * C$

L = the lower class boundary of the class containing the median

n = the total number of frequency

fm = frequency of the median class

Cfm = cumulative frequency of all the classes preceding the class containing the median

C = width of the class containing the median

MODE FROM A GROUPED DATA

- Mode = L + $\left[\frac{\Delta_1}{\Delta_1+\Delta_2}\right] * C$

L = the lower class boundary of the class containing the mode

Δ_1 = difference between the modal class frequency and the preceding class frequency

Δ_2 = difference between the modal class frequency and the succeeding (next) class frequency

C = width of the class containing the mode

EXAMPLE 3.5

Find the mode and median of the 40 male students at KSB by using the frequency distribution.

WEIGHT	FREQUENCY	CLASS BOUNDARIES	CUMULATIVE FREQUENCY
118 – 126	3		3
127 – 135	5		8
136 – 144	9		17
145 – 153	12		29
154 – 162	5		34
163 – 171	4		38
172 - 180	2		40